



Wednesday 18 June 2014 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

- 1 Express $\frac{3x}{(2-x)(4+x^2)}$ in partial fractions. [5]
- 2 Find the first three terms in the binomial expansion of $(4+x)^{\frac{3}{2}}$. State the set of values of x for which the expansion is valid. [5]
- 3 Fig. 3 shows the curve $y = x^3 + \sqrt{(\sin x)}$ for $0 \leq x \leq \frac{\pi}{4}$.

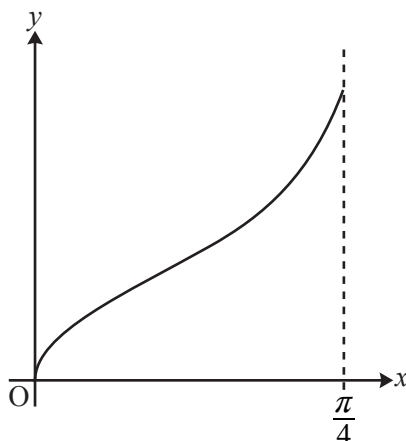


Fig. 3

- (i) Use the trapezium rule with 4 strips to estimate the area of the region bounded by the curve, the x -axis and the line $x = \frac{\pi}{4}$, giving your answer to 3 decimal places. [4]
- (ii) Suppose the number of strips in the trapezium rule is increased. Without doing further calculations, state, with a reason, whether the area estimate increases, decreases, or it is not possible to say. [1]
- 4 (i) Show that $\cos(\alpha + \beta) = \frac{1 - \tan \alpha \tan \beta}{\sec \alpha \sec \beta}$. [3]
- (ii) Hence show that $\cos 2\alpha = \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}$. [2]
- (iii) Hence or otherwise solve the equation $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \frac{1}{2}$ for $0^\circ \leq \theta \leq 180^\circ$. [3]
- 5 A curve has parametric equations $x = e^{3t}, y = te^{2t}$.
- (i) Find $\frac{dy}{dx}$ in terms of t . Hence find the exact gradient of the curve at the point with parameter $t = 1$. [4]
- (ii) Find the cartesian equation of the curve in the form $y = ax^b \ln x$, where a and b are constants to be determined. [3]

- 6 Fig. 6 shows the region enclosed by the curve $y = (1 + 2x^2)^{\frac{1}{3}}$ and the line $y = 2$.

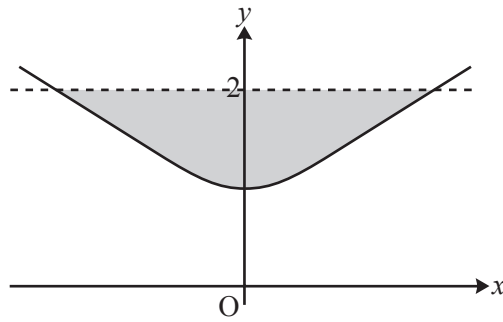


Fig. 6

This region is rotated about the y -axis. Find the volume of revolution formed, giving your answer as a multiple of π . [6]

Question 7 begins on page 4.

Section B (36 marks)

- 7 Fig. 7 shows a tetrahedron ABCD. The coordinates of the vertices, with respect to axes Oxyz, are $A(-3, 0, 0)$, $B(2, 0, -2)$, $C(0, 4, 0)$ and $D(0, 4, 5)$.

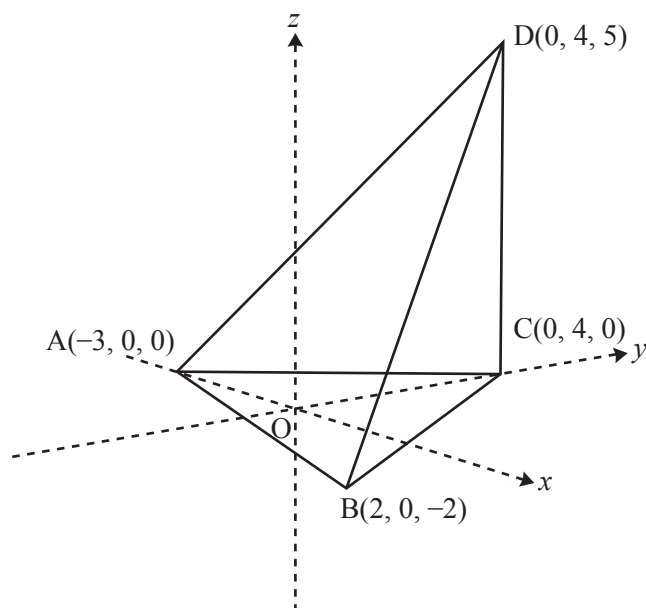


Fig. 7

- (i) Find the lengths of the edges AB and AC, and the size of the angle CAB. Hence calculate the area of triangle ABC. [7]
- (ii) (A) Verify that $4\mathbf{i} - 3\mathbf{j} + 10\mathbf{k}$ is normal to the plane ABC. [2]
 (B) Hence find the equation of this plane. [2]
- (iii) Write down a vector equation for the line through D perpendicular to the plane ABC. Hence find the point of intersection of this line with the plane ABC. [5]
- The volume of a tetrahedron is $\frac{1}{3} \times \text{area of base} \times \text{height}$.
- (iv) Find the volume of the tetrahedron ABCD. [2]

- 8 Fig. 8.1 shows an upright cylindrical barrel containing water. The water is leaking out of a hole in the side of the barrel.

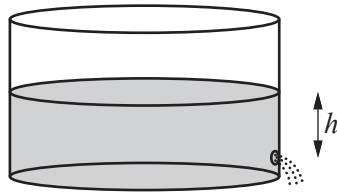


Fig. 8.1

The height of the water surface above the hole t seconds after opening the hole is h metres, where

$$\frac{dh}{dt} = -A\sqrt{h}$$

and where A is a positive constant. Initially the water surface is 1 metre above the hole.

- (i) Verify that the solution to this differential equation is

$$h = \left(1 - \frac{1}{2}At\right)^2. \quad [3]$$

The water stops leaking when $h = 0$. This occurs after 20 seconds.

- (ii) Find the value of A , and the time when the height of the water surface above the hole is 0.5 m. [4]

Fig. 8.2 shows a similar situation with a different barrel; h is in metres.

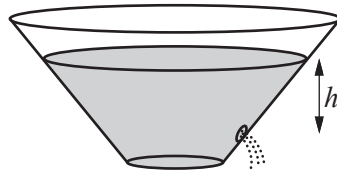


Fig. 8.2

For this barrel,

$$\frac{dh}{dt} = -B\frac{\sqrt{h}}{(1+h)^2},$$

where B is a positive constant. When $t = 0$, $h = 1$.

- (iii) Solve this differential equation, and hence show that

$$h^{\frac{1}{2}}(30 + 20h + 6h^2) = 56 - 15Bt. \quad [7]$$

- (iv) Given that $h = 0$ when $t = 20$, find B .

Find also the time when the height of the water surface above the hole is 0.5 m. [4]

END OF QUESTION PAPER

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A2 GCE MATHEMATICS (MEI)

4754/01 Applications of Advanced Mathematics (C4)

INSTRUCTIONS



The examination is in two parts:

Paper A (1 hour 30 minutes)

Paper B (up to 1 hour)

Supervisors are requested to ensure that Paper B **is not issued** until Paper A has been collected in from the candidates.

Centres may, if they wish, grant a supervised break between the two parts of this examination.

Paper B should not be attached to the corresponding paper A script. For Paper A only the candidates' printed answer books, in the same order as the attendance register, should be sent for marking; the question paper should be retained in the centre or recycled. For Paper B only the question papers, on which the candidates have written their answers, should be sent for marking; the insert should be retained in the centre or recycled. Any additional sheets used must be carefully attached to the correct paper.

For Paper B (Comprehension) only.

A standard English dictionary is allowed for the comprehension.

(Dictionaries to be used in the examination must be thoroughly checked before the examination.) Full regulations are in the JCQ Regulations and Guidance booklet.

This notice must be on the Invigilator's desk at all times during the morning of Wednesday 18 June 2014.

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PRINTED ANSWER BOOK

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- MEI Examination Formulae and Tables (MF2)

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- Scientific or graphical calculator

Duration: 1 hour 30 minutes



Candidate forename		Candidate surname	
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Centre number						Candidate number				
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Section A (36 marks)

1	

3 (i)	
3 (ii)	

4 (i)

4 (ii)

5 (i)

5 (ii)

7(ii)(A)

7(ii)(B)

7 (iii)	
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8 (iv)	



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Question	Answer	Marks	Guidance
1	$\frac{3x}{(2-x)(4+x^2)} = \frac{A}{2-x} + \frac{Bx+C}{4+x^2}$ $\Rightarrow 3x = A(4+x^2) + (Bx+C)(2-x)$ $x=2 \Rightarrow 6 = 8A, A = \frac{3}{4}$ $x^2 \text{ coeffs: } 0 = A - B \Rightarrow B = \frac{3}{4}$ $\text{constants: } 0 = 4A + 2C \Rightarrow C = -1\frac{1}{2}$	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[5]</p>	<p>correct form of partial fractions (condone additional coeffs eg $\frac{Ax+B}{2-x} + \frac{Cx+D}{4+x^2}$ * for M1 BUT $\frac{A}{2-x} + \frac{B}{4+x^2}$ ** is M0)</p> <p>Multiplying through oe and substituting values or equating coeffs at LEAST AS FAR AS FINDING A VALUE for one of their unknowns (even if incorrect) Can award in cases * and ** above Condone a sign error or single computational error for M1 but not a conceptual error Eg $3x = A(2-x) + (Bx+C)(4+x^2)$ is M0 $3x(2-x)(4+x^2) = A(4+x^2) + (Bx+C)(2-x)$ is M0 Do not condone missing brackets unless it is clear from subsequent work that they were implied. Eg $3x = A(4+x^2) + Bx + C(2-x) = 4A + Ax^2 + Bx + 2C - Cx$ is M0 $= 4A + Ax^2 + 2Bx - Bx^2 + 2C - Cx$ is M1</p> <p>oe www [SC B1 $A = 3/4$ from cover up rule can be applied, then the M1 applies to the other coefficients] NB $\frac{A}{2-x} + \frac{B}{4+x^2} \Rightarrow A = 3/4$ is A0 ww (wrong working)</p> <p>oe www</p> <p>oe www [In the case of * above, all 4 constants are needed for the final A1] Ignore subsequent errors when recompiling the final solution provided that the coeffs were all correct</p>

Question	Answer	Marks	Guidance
2	$(4+x)^{3/2} = 4^{3/2} \left(1 + \frac{x}{4}\right)^{3/2}$ $= 8 \left(1 + \frac{3}{2} \left(\frac{1}{4}x\right) + \frac{3}{2} \cdot \frac{1}{2} \cdot \frac{1}{2!} \left(\frac{1}{4}x\right)^2 + \dots\right)$ $= 8 + 3x$ $\qquad\qquad\qquad + 3/16 x^2$ <p>Valid for $-4 < x < 4$ or $x < 4$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>[5]</p>	<p>dealing with the '4' to obtain $4^{3/2} \left(1 + \frac{x}{4}\right)^{3/2}$</p> <p>(or expanding as $4^{3/2} + \frac{3}{2} 4^{1/2} x + \left(\frac{3}{2}\right) \left(\frac{1}{2}\right) 4^{-1/2} \frac{x^2}{2!} + \dots$ and having all the powers of 4 correct)</p> <p>correct binomial coeffs for $n = 3/2$ ie 1, 3/2, 3/2.1/2.1/2! Not nCr form Indep of coeff of x Indep of first M1</p> <p>8 + 3x www</p> <p>... + 3/16 x^2 www</p> <p>Ignore subsequent terms</p> <p>accept \leq s or a combination of $<$ and \leq, but not $-4 > x > 4$, $x > 4$, or say $-4 < x$ condone $-4 < x < 4$ Indep of all other marks</p> <p>Allow MR throughout this question for $n = m/2$ where $m \in \mathbb{N}$, and m odd and then -1 MR provided it is at least as difficult as the original.</p>

Question		Answer	Marks	Guidance												
3	(i)	<table border="1"> <tr> <td>x</td> <td>0</td> <td>0.1963</td> <td>0.3927</td> <td>0.5890</td> <td>0.7854</td> </tr> <tr> <td>y</td> <td>0</td> <td>0.4493</td> <td>0.6792</td> <td>0.9498</td> <td>1.3254</td> </tr> </table> $A = (\pi/32) [(0 + 1.3254) + 2(0.4493 + 0.6792 + 0.9498)]$ $= 0.538$	x	0	0.1963	0.3927	0.5890	0.7854	y	0	0.4493	0.6792	0.9498	1.3254	<p>B2,1,0</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>For values 0.4493,0.6792,0.9498 (4dp or better soi) [accept truncated to 4 figs after dec point]</p> <p>[cannot assume values of form $(\pi/16)^3 + \sqrt{(\sin \pi/16)}$ are correct unless followed by correct total at some later stage as some will be in degree mode]</p> <p>Use of the trapezium rule. Trapezium rule formula for 4 strips must be seen, with or without substitution seen. Correct h must be soi. [accept separate trapezia added]</p> <p>0.538 www 3dp only (NB using 1.325 is ww)</p> <p>SC B0 0.538 without any working as no indication of strips or method used</p> <p>SC B1 0.538 with some indication of 4 strips but no values seen</p> <p>Correct values followed by 0.538 scores B2 B0</p> <p>Correct values followed by correct formula for 4 strips, with or without substitution seen, then $A= 0.538$ scores 4/4.</p> <p>Correct formula for 4 strips and values of form $((\pi/16)^3 + \sqrt{(\sin \pi/16)})\dots$ followed by correct answer scores 4/4 (or $\frac{3}{4}$ with wrong dp)</p> <p>NB Values given in the table to only 3dp give apparently the correct answer, but scores B0,M1A0 ww</p>
x	0	0.1963	0.3927	0.5890	0.7854											
y	0	0.4493	0.6792	0.9498	1.3254											
3	(ii)	Not possible to say, eg some trapezia are above and some below curve oe.	B1	Need a reason. Must be without further calculation.												
			[1]													

Question	Answer	Marks	Guidance
4 (i)	<p>EITHER Use of $\cos = 1/\sec$ (or $\sin = 1/\operatorname{cosec}$)</p> <p>From RHS</p> $\frac{1 - \tan \alpha \tan \beta}{\sec \alpha \sec \beta}$ $= \frac{1 - \sin \alpha / \cos \alpha \cdot \sin \beta / \cos \beta}{1 / \cos \alpha \cdot 1 / \cos \beta}$ $= \cos \alpha \cos \beta \left(1 - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}\right)$ $= \cos \alpha \cos \beta - \sin \alpha \sin \beta$ $= \cos(\alpha + \beta)$	<p>B1</p> <p>M1</p> <p>A1</p>	<p>Must be used</p> <p>Substituting and simplifying as far as having no fractions within a fraction</p> <p>[need more than $\frac{1-tt}{\sec \sec} = cc - ss$ ie an intermediate step that can lead to cc-ss]</p> <p>Convincing simplification and correct use of $\cos(\alpha + \beta)$</p> <p>Answer given</p>
	<p>OR From LHS, $\cos = 1/\sec$ or $\sin = 1/\operatorname{cosec}$ used</p> $\cos(\alpha + \beta)$ $= \cos \alpha \cos \beta - \sin \alpha \sin \beta$ $= \frac{1}{\sec \alpha \sec \beta} - \sin \alpha \sin \beta$ $= \frac{1 - \sec \alpha \sin \alpha \sec \beta \sin \beta}{\sec \alpha \sec \beta}$ $= \frac{1 - \tan \alpha \tan \beta}{\sec \alpha \sec \beta}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>Correct angle formula and substitution and simplification to one term</p> <p>OR eg $\cos \alpha \cos \beta - \sin \alpha \sin \beta$ $= \cos \alpha \cos \beta (1 - \tan \alpha \tan \beta)$</p> <p>Simplifying to final answer www</p> <p>Answer given</p> <p>Or any equivalent work but must have more than cc-ss = answer.</p>

Question		Answer	Marks	Guidance
4	(ii)	$\beta = \alpha$ $\cos 2\alpha = \frac{1 - \tan^2 \alpha}{\sec^2 \alpha}$ $= \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}$	M1	$\beta = \alpha$ used , Need to see $\sec^2 \alpha$
		A1	Use of $\sec^2 \alpha = 1 + \tan^2 \alpha$ to give required result Answer Given	
		OR, without Hence, $\cos 2\alpha = \cos^2 \alpha \left(1 - \frac{\sin^2 \alpha}{\cos^2 \alpha}\right)$ $= \frac{1}{\sec^2 \alpha} (1 - \tan^2 \alpha)$ $= \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}$	M1	Use of $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$ soi Simplifying and using $\sec^2 \alpha = 1 + \tan^2 \alpha$ to final answer Answer Given Accept working in reverse to show RHS=LHS, or showing equivalent
			[2]	
4	(iii)	$\cos 2\theta = \frac{1}{2}$ $2\theta = 60^\circ, 300^\circ$ $\theta = 30^\circ, 150^\circ$	M1	Soi or from $\tan^2 \theta = 1/3$ oe from $\sin^2 \theta$ or $\cos^2 \theta$
			A1	First correct solution
			A1	Second correct solution and no others in the range SC B1 for $\pi/6$ and $5\pi/6$ and no others in the range
			[3]	

Question		Answer	Marks	Guidance
5	(i)	EITHER $x = e^{3t}, y = te^{2t}$ $dy / dt = 2te^{2t} + e^{2t}$ $\Rightarrow dy/dx = (2te^{2t} + e^{2t})/3e^{3t}$ when $t = 1, dy/dx = 3e^2/3e^3 = 1/e$	B1 M1 A1 A1	soi Their $dy / dt \div dx / dt$ in terms of t oe cao allow for unsimplified form even if subsequently cancelled incorrectly ie can isw cao www must be simplified to $1/e$ oe
		OR $3t = \ln x, y = \frac{\ln x}{3} e^{2/3 \ln x} = \frac{x^{2/3} \ln x}{3}$ $dy / dx = \frac{1}{3} x^{2/3} \frac{1}{x} + \ln x \frac{2}{9} x^{-1/3}$ $= \frac{1}{3e^t} + \frac{2t}{3e^t}$ $dy/dx = 1/3e + 2/3e = 1/e$	B1 M1 A1 A1 [4]	Any equivalent form of y in terms of x only Differentiating their y provided not eased ie need a product including $\ln kx$ and x^p and subst $x = e^{3t}$ to obtain dy/dx in terms of t oe cao www cao exact only must be simplified to $1/e$ or e^{-1}
5	(ii)	$3t = \ln x \Rightarrow t = (\ln x)/3$ $y = (\ln x) / 3e^{(2 \ln x)/3}$ $y = \frac{1}{3} x^{\frac{2}{3}} \ln x$	B1 M1 A1 [3]	Finding t correctly in terms of x Subst in y using their t Required form $ax^b \ln x$ only NB If this work was already done in 5(i), marks can only be scored in 5(ii) if candidate specifically refers in this part to their part (i).

Question	Answer	Marks	Guidance
6	$y = (1 + 2x^2)^{\frac{1}{3}} \Rightarrow y^3 = 1 + 2x^2$ $\Rightarrow x^2 = \frac{1}{2}(y^3 - 1)$ $V = \int_1^2 \pi x^2 dy = \frac{1}{2} \pi \int_1^2 (y^3 - 1) dy$ $= \frac{1}{2} \pi \left[\frac{1}{4} y^4 - y \right]_1^2 = \frac{1}{2} \pi \left(2 + \frac{3}{4} \right)$ $= \frac{11}{8} \pi$	<p>M1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[6]</p>	<p>finding x^2 (or x) correctly in terms of y</p> <p>For M1 need $\int \pi x^2 dy$ with substitution for their x^2 (in terms of y only)</p> <p>Condone absence of dy throughout if intentions clear. (need π)</p> <p>www For A1 it must be correct with correct limits 1 and 2, but they may appear later</p> <p>$1/2[y^4/4 - y]$ independent of π and limits</p> <p>substituting both their limits in correct order in correct expression, condone a minor slip for M1 (if using $y = 0$ as lower limit then '-0' is enough)</p> <p>condone absence of π for M1</p> <p>oe exact only www ($1\frac{3}{8} \pi$ or 1.375π)</p>

Question	Answer	Marks	Guidance
7 (i)	$AB = \sqrt{5^2 + (-2)^2} = \sqrt{29}$ $AC = \sqrt{3^2 + 4^2} = 5$ $\cos \theta = \frac{\begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}}{\sqrt{29} \cdot 5} = \frac{15+0+0}{5\sqrt{29}} = 0.5571$ $\Rightarrow \theta = 56.15^\circ$ $\text{Area} = \frac{1}{2} \times 5 \times \sqrt{29} \times \sin \theta$ $= 11.18$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[7]</p>	<p>5.39 or better (condone sign error in vector for B1)</p> <p>Accept $\sqrt{25}$ (condone sign error in vector for B1)</p> <p>$\cos \theta = \frac{\text{scalar product of AB with AC}}{ \text{AB} \cdot \text{AC} }$ (accept BA/CA)</p> <p>with substitution condone a single numerical error provided method is clearly understood [OR Cosine Rule, as far as $\cos \theta =$ correct numerical expression]</p> <p>$\text{www} \pm 0.5571, 0.557, 15/5\sqrt{29}, 15/\sqrt{25}\sqrt{29}$ oe or better soi (\pm for method only)</p> <p>www Accept answers that round to 56.1° or 56.2° or 0.98 radians (or better)</p> <p>NB vector $5\mathbf{i}+0\mathbf{j}+2\mathbf{k}$ leads to apparently correct answer but loses all A marks in part(i)</p> <p>Using their AB, AC, \angle CAB. Accept any valid method using trigonometry</p> <p>Accept $5\sqrt{5}$ and answers that round to 11.18 or 11.19 (2dp) www or SCA1 for accurate work soi rounded at the last stage to 11.2 (but not from an incorrect answer, say from an incorrect angle or from say 11.17 or 11.22 stated and rounded to 11.2) We will not accept inaccurate work from over rounded answers for the final mark.</p>

Question			Answer	Marks	Guidance
7	(ii)	(A)	$\overline{AB} \cdot \begin{pmatrix} 4 \\ -3 \\ 10 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -3 \\ 10 \end{pmatrix} = 5 \cdot 4 + 0 \cdot (-3) + (-2) \cdot 10 = 0$ $\overline{AC} \cdot \begin{pmatrix} 4 \\ -3 \\ 10 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -3 \\ 10 \end{pmatrix} = 3 \times 4 + 4 \times (-3) + 0 \times 10 = 0$	<p>B1</p> <p>B1</p> <p>[2]</p>	<p>Scalar product with one vector in the plane with numerical expansion shown.</p> <p>Scalar product, as above, with evaluation, with a second vector. NB vectors are not unique</p> <p>SCB2 finding the equation of plane first by any valid method (or using vector product) and then clearly stating that the normal is proportional to the coefficients.</p> <p>SC For candidates who substitute all three points in the plane $4x-3y+10z = c$ and show that they give the same result, award M1 If they include a statement explaining why this means that $4\mathbf{i}-3\mathbf{j}+10\mathbf{k}$ is normal they can gain A1.</p>
7	(ii)	(B)	$4x - 3y + 10z = c$ $\Rightarrow 4x - 3y + 10z + 12 = 0$	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>Required form and substituting the co-ordinates of a point on the plane</p> <p>oe If found in (A) it must be clearly referred to in (B) to gain the marks. Do not accept vector equation of the plane, as 'Hence'.</p> <p>$4\mathbf{i}-3\mathbf{j}+10\mathbf{k} = -12$ is M1A0</p>

Question		Answer	Marks	Guidance
7	(iii)	$\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix}$ $+ \lambda \begin{pmatrix} 4 \\ -3 \\ 10 \end{pmatrix}$ <p>Meets $4x - 3y + 10z + 12 = 0$ when $16\lambda - 3(4 - 3\lambda) + 10(5 + 10\lambda) + 12 = 0$ $\Rightarrow 125\lambda = -50, \lambda = -0.4$ So meets plane ABC at $(-1.6, 5.2, 1)$</p>	B1 B1 M1 A1 A1 [5]	Need $\mathbf{r} =$ (or $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$) oe Subst their $4\lambda, 4 - 3\lambda, 5 + 10\lambda$ in equation of their plane from (ii) $\lambda = -0.4$ (NB not unique) cao www (condone vector)
7	(iv)	$\text{height} = \sqrt{(1.6^2 + (-1.2)^2 + 4^2)} = \sqrt{20}$ $\text{volume} = 11.18 \times \sqrt{20} / 3 = 16.7$	B1ft B1cao [2]	ft their (iii) 50/3 or answers that round to 16.7 www and not from incorrect answers from (iii) ie not from say (1.6, 2.8, 9)

Question		Answer	Marks	Guidance
8	(i)	<p>Either $h = (1 - \frac{1}{2} At)^2 \Rightarrow dh/dt = -A(1 - \frac{1}{2} At)$ $= -A\sqrt{h}$ when $t = 0, h = (1 - 0)^2 = 1$ as required OR $\int \frac{dh}{\sqrt{h}} = \int -A dt$ $2h^{1/2} = -At + c$ $h = \left(\frac{-At + c}{2}\right)^2$ at $t=0, h = 1, 1 = (c/2)^2 \Rightarrow c = 2, h = (1 - At/2)^2$</p>	<p>M1 A1 B1 M1 A1 B1 [3]</p>	<p>Including function of a function, need to see middle step AG Separating variables correctly and integrating Including c. [Condone change of c.] Using initial conditions AG</p>
8	(ii)	<p>When $t = 20, h = 0$ $\Rightarrow 1 - 10A = 0, A = 0.1$ When the depth is 0.5 m, $0.5 = (1 - 0.05t)^2$ $\Rightarrow 1 - 0.05t = \sqrt{0.5}, t = (1 - \sqrt{0.5})/0.05 = 5.86s$</p>	<p>M1 A1 M1 A1 [4]</p>	<p>Subst and solve for A cao substitute $h = 0.5$ and their A and solve for t www cao accept 5.9</p>

Question	Answer	Marks	Guidance
8 (iii)	$\frac{dh}{dt} = -B \frac{\sqrt{h}}{(1+h)^2}$ $\Rightarrow \int \frac{(1+h)^2}{\sqrt{h}} dh = - \int B dt$	M1	separating variables correctly and intend to integrate both sides (may appear later) [NB reading $(1+h)^2$ as $1+h^2$ eases the question. Do not mark as a MR] In cases where $(1+h)^2$ is MR as $1+h^2$ or incorrectly expanded, as say $1+h+h^2$ or $1+h^2$, allow first M1 for correct separation and attempt to integrate and can then score a max of M1M0A0A0A1 (for $-Bt+c$) A0A0, max 2/7.
	<p>EITHER, LHS</p> $\int \frac{1+2h+h^2}{\sqrt{h}} dh$ $= \int (h^{-1/2} + 2h^{1/2} + h^{3/2}) dh$	M1 A1	expanding $(1+h)^2$ and dividing by \sqrt{h} to form a one line function of h (indep of first M1) with each term expressed as a single power of h eg must simplify say $1/\sqrt{h}+2h/\sqrt{h}+h^2/\sqrt{h}$, condone a single error for M1 (do not need to see integral signs) $h^{-1/2} + 2h^{1/2} + h^{3/2}$ cao dep on second M only -do not need integral signs
	<p>OR, LHS, EITHER</p> $(1+2h+h^2)2h^{1/2} - \int 2h^{1/2}(2+2h)dh$ <p>OR</p> $h^{1/2} + h^{3/2} + \frac{h^{5/2}}{3} + \int \frac{1}{2} h^{-3/2} (h+h^2 + \frac{h^3}{3}) dh$	M1 A1	using $\int u dv = uv - \int v du$ correct formula used correctly, indep of first M1 condone a single error for M1 if intention clear cao oe
	$2h^{1/2} + \frac{4h^{3/2}}{3} + \frac{2h^{5/2}}{5}$ $= -Bt + c$ $\Rightarrow 2h^{1/2} + 4h^{3/2}/3 + 2h^{5/2}/5 = -Bt + c$ <p>When $t = 0, h = 1 \Rightarrow c = 56/15$</p> $\Rightarrow h^{1/2}(30 + 20h + 6h^2) = 56 - 15Bt \quad *$	A1 A1 A1 A1 [7]	cao oe, both sides dependent on first M1 mark cao need $-Bt$ and c for second A1 but the constant may be on either side from correct work only (accept 3.73 or rounded answers here but not for final A1) or $c = -56/15$ if constant on opposite side. NB AG must be from all correct exact work including exact c.

Question		Answer	Marks	Guidance
8	(iv)	$h = 0$ when $t = 20$ $\Rightarrow B = 56/300 = 0.187$ When $h = 0.5$ $56 - 2.8t = 29.3449\dots$ $\Rightarrow t = 9.52s$	M1 A1 M1 A1 [4]	Substituting $h = 0, t = 20$ Accept 0.187 Subst their $h = 0.5$, ft their B and attempt to solve Accept answers that round to 9.5s www.



Wednesday 18 June 2014 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01B Applications of Advanced Mathematics (C4) Paper B: Comprehension

QUESTION PAPER

Candidates answer on the Question Paper.

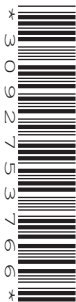
OCR supplied materials:

- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator
- Rough paper

Duration: Up to 1 hour



Candidate forename		Candidate surname	
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Centre number						Candidate number				
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INSTRUCTIONS TO CANDIDATES

- The Insert will be found inside this document.
- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- The insert contains the text for use with the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.
- This document consists of **8** pages. Any blank pages are indicated.

- 1 The example illustrated in Table 1 in the article referred to the members of a racing club.

This question is about a similar club and the equivalent table is given below. The club has 150 members and each of them belongs to one of the groups P, Q, R or S.

For each group the probability of an accident and the average cost is given in the table below.

Group	P	Q	R	S
Proportion of people	10%	20%	40%	30%
Probability of an accident	0.1	0.15	0.05	0.2
Average cost per accident	£5000	£2000	£1000	£500

- (i) Complete the table below. [3]
- (ii) Calculate the premium if each member of the club pays the same amount and the total of the premiums is equal to the total average cost of accidents per year. [2]
- (iii) The club management decides that the total of the premiums should be 50% more than the total average cost of accidents per year, and that each group should pay an amount proportional to its contribution to the total average cost.

Calculate the premium for a member of group S.

[2]

1 (i)	<table border="1"> <thead> <tr> <th>Group</th> <th>P</th> <th>Q</th> <th>R</th> <th>S</th> </tr> </thead> <tbody> <tr> <td>Number of people</td> <td>15</td> <td></td> <td></td> <td></td> </tr> <tr> <td>Average number of accidents in a year</td> <td></td> <td>4.5</td> <td></td> <td></td> </tr> <tr> <td>Average cost of accidents per year</td> <td>£7500</td> <td>£9000</td> <td></td> <td></td> </tr> </tbody> </table>	Group	P	Q	R	S	Number of people	15				Average number of accidents in a year		4.5			Average cost of accidents per year	£7500	£9000		
	Group	P	Q	R	S																
	Number of people	15																			
	Average number of accidents in a year		4.5																		
	Average cost of accidents per year	£7500	£9000																		

1 (ii)	
1 (iii)	

PLEASE DO NOT WRITE IN THIS SPACE

2 In lines 97 to 100, the article says

“Most insurance companies have a maximum no-claims discount of 65%. One way of interpreting this practice is that the figure arrived at by applying the maximum no-claims discount is actually the basic cost of the insurance, and that drivers who have not earned the so-called discount are actually paying a surcharge.”

A new driver without any no-claims discount pays k times “the basic cost of the insurance”.

Find the value of k .

[2]

2	

PLEASE DO NOT WRITE IN THIS SPACE

3 On June 1st 2007 Louise paid her first car insurance premium. She did not have any no-claims discount.

She retained her policy with the same insurance company. She had no accidents and so did not make any claim on her insurance. Her no-claims discount followed the pattern in Table 7. Apart from her no-claims discount, her basic premium remained the same.

On May 31st 2014, Louise calculated that so far her no-claims discount had saved her £3875.

What premium did Louise pay on June 1st 2007?

[3]

3	

4 In lines 68 to 69, the article says

“Clearly the role of inexperience goes down with age. A possible mathematical model is that it decays exponentially.”

This question investigates this in the case of male drivers. The relevant part of Table 4 is reproduced here.

Age (in years)	18.0	20.5	23.0	27.5	45.0
Male %	36	15	6	2.5	1

A model is proposed in which

$$y = a + be^{-k(x-17)},$$

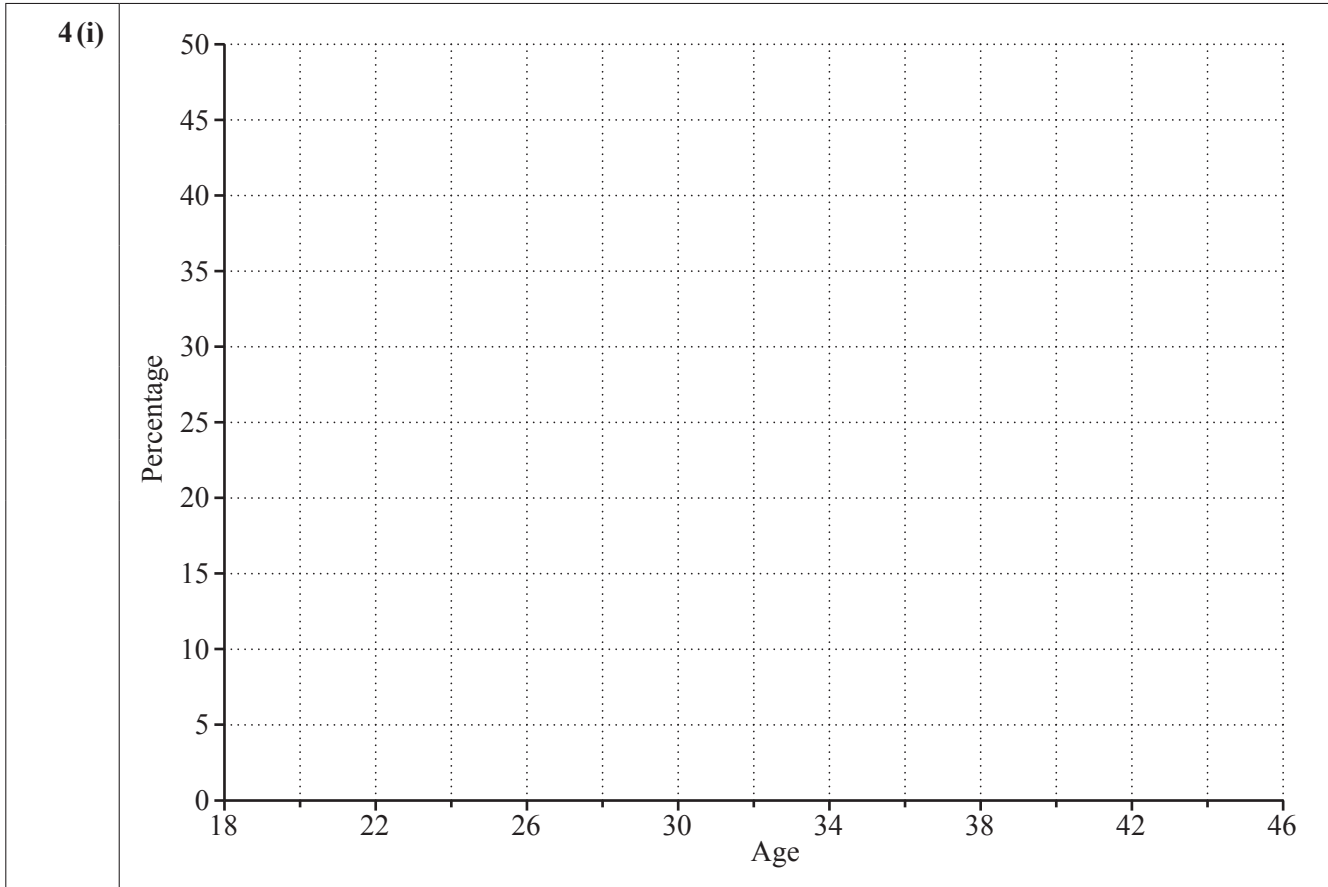
where

- x is the age of the driver,
- y is the percentage of accidents in which inexperience played a role,
- a , b and k are constants to be determined.

(i) Draw a sketch graph of y against x for values of x between 18 and 45. [1]

(ii) Explain why the graph justifies putting $a = 1$. [1]

(iii) Taking $a = 1$ and using the first two data points, it is estimated that $b = 50.5$ and $k = 0.3665$. Investigate whether these values are consistent with the point (23, 6). [2]



4 (ii)	
4 (iii)	

- 5 In this question the number of points on a driver's licence for motoring offences is denoted by n .

When calculating a driver's premium, a particular insurance company takes account of such offences by multiplying the premium by an amount M , where

$$M = 1 \quad \text{if } n \leq 3,$$

$$M = 2^{\frac{n}{6}} \quad \text{if } 3 < n < 12.$$

A driver who is insured with this company is paying a premium of £520. Initially he has no points on his licence.

He is convicted of a speeding offence and so receives 3 points on his licence. He is then convicted for dangerous driving and receives a further 6 points on his licence.

Calculate his premium

(A) after his first offence,

(B) after his second offence.

[2]

5 (A)	
5 (B)	



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Wednesday 18 June 2014 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01B Applications of Advanced Mathematics (C4) Paper B: Comprehension

INSERT

Duration: Up to 1 hour



INFORMATION FOR CANDIDATES

- This Insert contains the text for use with the questions.
- This document consists of 8 pages. Any blank pages are indicated.

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Insurance for young drivers

Introduction

Most people would regard being able to drive as an essential skill for adults in today's world. Passing the driving test is an important event for many young people; some see it as a passport into adult life.

The test is not particularly easy but passing it is often only the start of your worries. For example, you then have to pay for motor insurance and this can be very expensive for young inexperienced drivers. This article looks at why insurance costs are so high, and what you can do about it.

5

Insurance: a simple model

Insurance is about sharing the cost associated with risk.

Imagine, for example, that 100 people belong to a racing club; they have regular races using a particular type of vehicle. They provide their own vehicles. On average, 10 people have an accident each year and the cost of repairs per accident is £5000. So, in any year there is a probability of $\frac{1}{10}$ that a particular person will have an accident. The members of the club agree to share the risk by each paying an annual amount of $\frac{1}{10} \times £5000 = £500$ into a fund to provide enough money to compensate those who have accidents. This simplified example illustrates the idea of sharing risk which underpins insurance; the £500 payment is called the *premium*.

10

15

Insurance is virtually always run by companies and they add an amount to any premium to cover their own costs, and to allow them a profit margin. They also need to ensure that they have sufficient money to cover a bad run of accidents. These factors are not taken into account in this example.

Refining the model

20

In the example above, everyone was assigned an equal probability of an accident of $\frac{1}{10}$ and paid the same premium. In practice, insurance companies try to identify high-risk groups and charge them higher premiums, and low-risk groups who can pay lower premiums. They also vary premiums according to the likely cost of an accident. Thus expensive and fast cars tend to attract higher premiums.

Imagine that, in the example of the racing club, three distinct groups of people, denoted by A, B and C, can be identified. Table 1 gives the proportion of people in each of these groups, the probabilities of their having an accident in any year and the average cost of an accident.

25

Group	A	B	C
Proportion of people	10%	30%	60%
Probability of an accident	0.4	0.1	0.05
Average cost per accident	£9500	£3000	£1000

Table 1 Probabilities (per year) and average costs of accidents for different groups

So among the 100 people in the club, 10 are in Group A, 30 in Group B and the rest in Group C.

The average number of accidents per year is 4 for those in Group A, 3 for those in Group B and 3 for those in Group C.

30

The average cost of these accidents is £38 000 for Group A, £9000 for Group B and £3000 for Group C, making £50 000 in total. So, although only 10% of the people are in Group A, on average they account for 76% of the total accident costs.

In fact, the average cost per year of the accidents per person in Group A is £3800; in Group B it is £300 and in Group C it is £50. 35

Clearly it would not be fair for those in the three groups to pay the same premiums; those in Group A should pay very much more.

Insurance for car drivers

Although the example of the racing club, with just three groups, is very much simpler than a real-life situation, it does highlight the two basic elements involved in calculations of insurance premiums: the probability and the cost of an accident. 40

In the example, the premium was determined by sharing the estimated cost of accidents among a specific group of people. In real motor insurance, it is not possible to define such a closed group, and so premiums are calculated purely on the basis of estimates of the probabilities and costs of accidents.

In motor insurance, important considerations when estimating the probability of an accident include the driver's age, gender, experience and past record. Those for the cost of an accident include the power of the car (and so its speed) and its value. 45

The driver's age and gender

The age of the driver is an important factor in determining the risk. Table 2, which is based on data from the Association of British Insurers relating to serious accidents, compares the accident rates of drivers, by age group and by gender. The group with the lowest rate, female drivers aged 60 to 69, has been assigned a value of 1, and all the others are compared to that. Thus a male driver in the 17 to 20 age group is 13.4 times as likely to have a serious accident as a female in the 60 to 69 age group. 50

In the rest of this article it is assumed that this general pattern can be applied to all accidents that give rise to insurance claims. 55

Age group	17–20	21–29	30–39	40–59	60–69	70+
Male	13.4	4.8	2.9	1.9	1.2	1.4
Female	11.3	4.6	2.8	1.9	1.0	1.2

Table 2 Relative accident rates for car drivers, classified by age group and by gender
(Source ABI)

Table 2 shows that young drivers are much more likely to be involved in accidents than older drivers; figures like these make it inevitable that insurance premiums for young drivers are high. Table 2 also shows that the accident rate for male drivers is higher than that for females for most age groups.

Some people start learning to drive on the road on their 17th birthday, the first possible opportunity, whereas others wait until they are quite a lot older. So while years of driving experience and age are related, the relationship is not a simple one. Table 3 shows the age and gender distribution of those holding full UK driving licences; figures are given in thousands, to the nearest 1000; it does not include those with provisional licences, many of whom are in the 17 to 20 age group.

60

Age group	17–20	21–29	30–39	40–59	60–69	70+
Male	538	2444	3430	8277	3467	2333
Female	482	2200	3037	7109	2726	1499

Table 3 Distribution by age and gender of UK licence holders ($\times 1000$) (Source DVLA)

Table 3 shows that in all age groups male drivers outnumber female drivers.

The most striking feature of the figures in Table 2 is the difference in accident rate according to age. An obvious cause is inexperience of driving. Table 4 gives estimates from the insurance industry of the percentage of accidents to which inexperience contributes, for drivers of certain ages.

65

Age (in years)	18.0	20.5	23.0	27.5	45.0
Male %	36	15	6	2.5	1
Female %	39	18.5	7.5	3.5	1.5

Table 4 Estimated role of inexperience in accidents (Source ABI)

Clearly the role of inexperience goes down with age. A possible mathematical model is that it decays exponentially.

Other data show that young drivers are more likely to have accidents at night and this is particularly so for males. One possible explanation for this is lack of experience of driving in the dark; if so, the driving test could be changed so as to include some night-time driving but that might not be easy to implement. Another explanation is that the drivers have been at parties; the worst times for accidents are Friday and Saturday nights.

70

While inexperience is obviously an important consideration, it clearly is not the only cause. Fig. 5 provides another part of the explanation. It shows the estimated percentages of accidents in which excessive speed is a contributory factor. There are large differences between male and female drivers for all the age groups concerned, particularly among the youngest drivers.

75

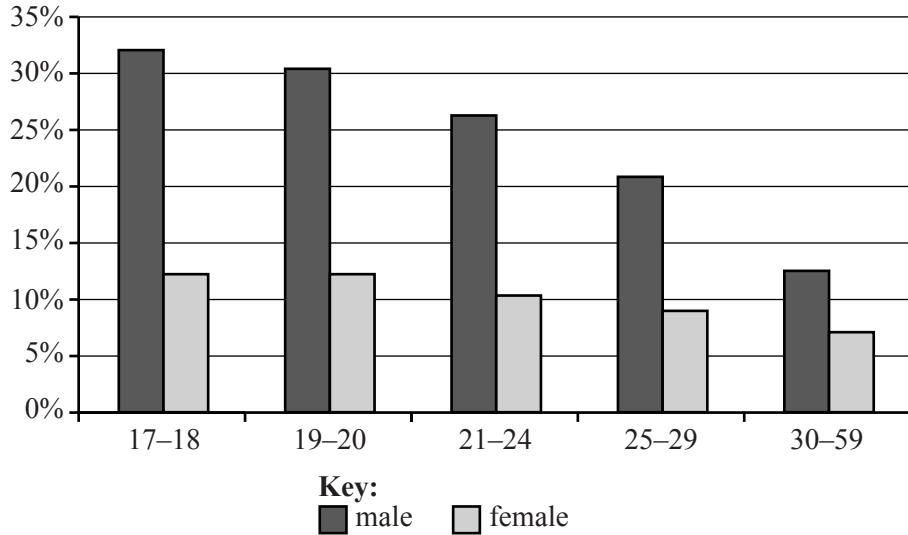


Fig. 5 Percentage of accidents with excessive speed as a contributory factor (Source DfT, STATS19)

The faster you are driving at the time of an accident, the more serious the accident is likely to be and so the greater the claim on the insurance company. Fig. 6 shows the average costs of claims for different groups of drivers. They are highest for young drivers; for almost all age groups they are higher for males than females.

80

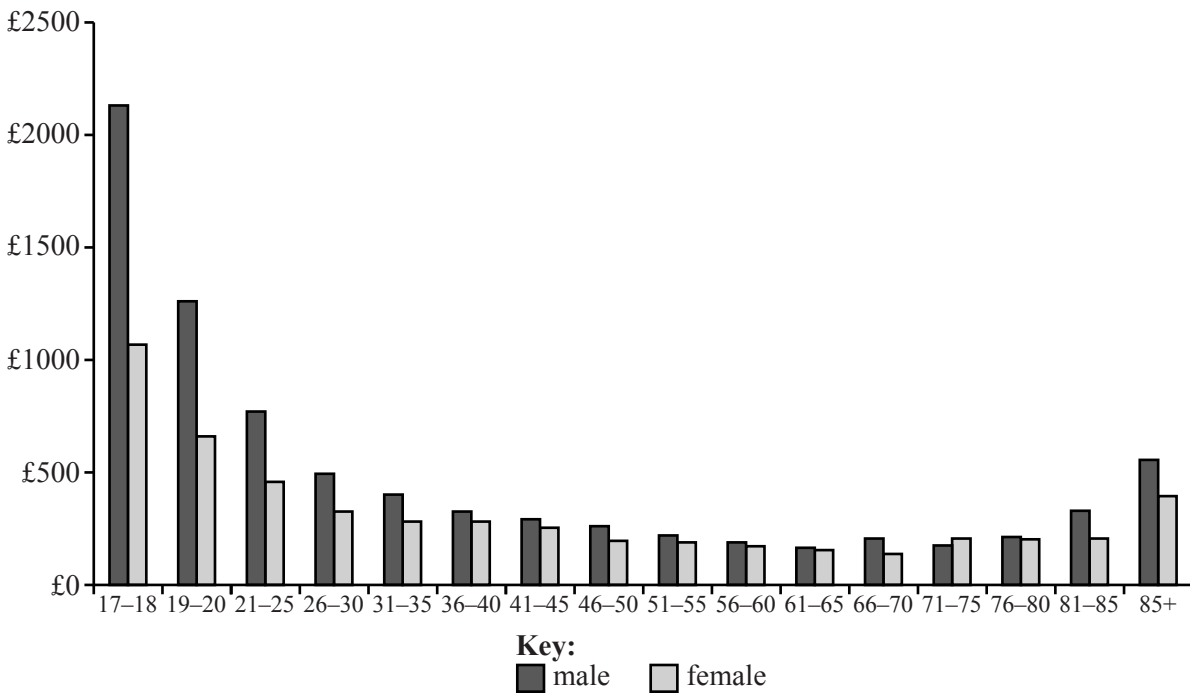


Fig. 6 Average costs of claims for different groups (Data source DfT, 2009)

Reducing insurance costs

The two major considerations when an insurance premium is being calculated are the probability of an accident and the average cost if it happens. Both of these are highest for young drivers and higher for males than females. 85

So what can young drivers do to reduce their insurance premiums?

Probability of an accident

When you are first insured, a company has no information about how careful a driver you will be, and so it is assumed that you will be as likely to have an accident as anyone else in your situation. It is up to you to prove otherwise. If you avoid accidents, the insurance company will usually reduce your future premiums. 90

Nearly all insurance companies reduce the premium of a driver who does not make any claims. This reduction is called a *no-claims discount*. Reductions vary from one insurance company to another but typical values are shown in Table 7. A good no-claims discount saves a lot of money. A no-claims discount has to be earned by claim-free driving and so is never given to a new driver. 95

Years of claim-free driving	% discount from the full premium
1	30%
2	40%
3	50%
4	60%
5+	65%

Table 7 Typical no-claims discounts

Most insurance companies have a maximum no-claims discount of 65%. One way of interpreting this practice is that the figure arrived at by applying the maximum no-claims discount is actually the basic cost of the insurance, and that drivers who have not earned the so-called discount are actually paying a surcharge. 100

If you are a very careful young driver it may seem unfair that you have to pay the same insurance premium as others who are reckless; you are paying extra for their bad driving. One way to prove that you really are a good driver is to make use of *telematics technology*. A small device is fitted to your car and transmits information to your insurance company, typically covering five aspects of your driving: cornering, swerving, braking, speed and acceleration. Feedback is provided in the form of advice on how to improve your driving and so reduce your insurance premium. 105

There is no discount for not breaking the law but if you are convicted for an offence like speeding or using your mobile phone while driving, an insurance company will see this as evidence of bad driving and so will probably increase your premium. Convictions result in points on your licence on a scale of 0 (a clean licence) to 12 (the level at which you are normally disqualified from driving). 110

Another risk factor is the number of passengers in the car. The figures in Table 8 show that the percentage of serious accidents increases very greatly with the number of passengers drivers are carrying in their cars. Among the possible explanations are that drivers are distracted or that they try to show off.

Number of passengers	0	1	2	3+
% of serious accidents	14	20	26	40

Table 8 Serious accidents classified according to the number of passengers in the car

The data in this article show that young female drivers have a lower probability of accidents than young male drivers; so you would expect lower premiums for females. Until recently that was the case, but a ruling of the European Court of Justice has now made this illegal.

115

Cost of an accident

The cost of an accident is obviously likely to increase with the value of the car. Not only will an expensive car be more expensive to repair, but it is likely to be more powerful and so able to go faster.

Insurance companies place cars in bands between 1 and 50 with the most expensive in the groups with the highest numbers. The lower the band of car you have, the cheaper the premium. Table 9 lists the insurance groups for a few new cars at the time of writing this article.

120

Car	Insurance group
Skoda Citigo 5 1.0	1
Peugeot 107 Access 1.0	3
Fiat 500 1.2 Pop	5
Nissan NOTE 1.4 Acenta	10
Volvo S40 D2 ES	20
Renault Clio Sport 200	30
Chrysler 300C Executive	40
Jaguar XKR 5.0 Coupé	50

Table 9 Insurance groups for some new cars

Calculating insurance premiums

Each insurance company has its own way of calculating a driver's premium. That is why the premium varies from one company to another. A company has to strike a balance between two different demands.

125

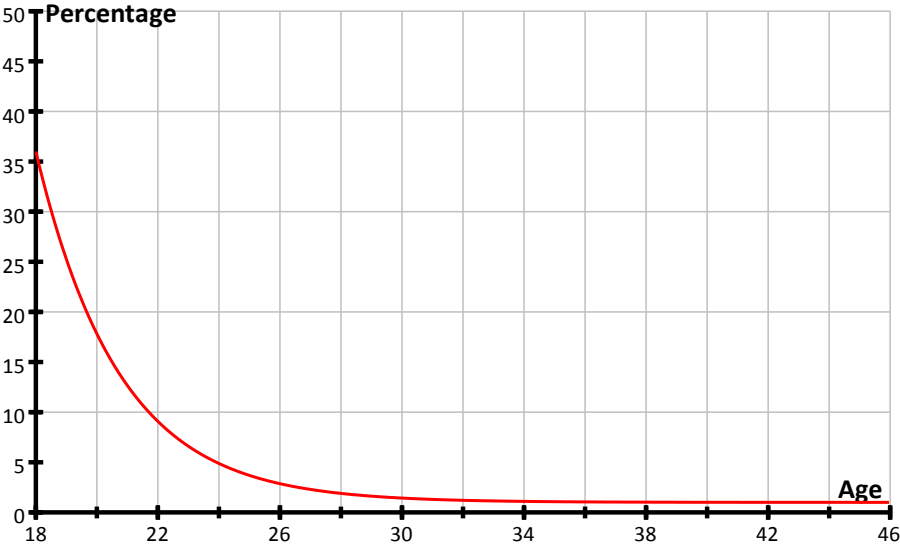
- The premium must be low enough to be attractive to customers.
- It must be high enough to ensure the company does not make a long-term loss.

In addition insurance companies offer benefits, such as a reduced premium in exchange for agreement to pay the first £200 (say) of any claim, so the premium is not the only consideration when choosing between companies.

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Question		Answer					Marks	Guidance
1	(i)						B1	for all three entries 30,60,45 correct
		Group	P	Q	R	S	B1	for all three entries 1.5, 3, 9 correct
		Number of people	15	30	60	45	B1	for 3000,4500 both correct
		Average number of accidents in a year	1.5	4.5	3	9		SC B2 for all entries in any three columns correct
		Average cost of accidents per year	£7500	£9000	£3000	£4500		
						[3]		
1	(ii)	$\text{£}24000 \div 150$ $=\text{£}160$					M1 A1 [2]	Adding their bottom row (7500 + 9000 + '3000' + '4500' = '24000' and dividing by 150 soi (and not divided or multiplied by any additional values) ft their 24000 ÷ 150 (corr to 2dp if inexact)
1	(iii)	The 45 members of Group S pay $1.5 \times \text{£}4500 = \text{£}6750$. So each pays $\text{£}6750 \div 45$ $=\text{£}150$.					M1 A1 [2]	their 4500×1.5 oe soi as part of solution cao www (must be from correct final column)

Question	Answer	Marks	Guidance																		
2	Basic premium = 35% of driver's premium Drivers premium = Basic premium ÷ 0.35 = 2.86 Basic premium ⇒ k = 2.86	M1 A1 [2]	use of 35% or 0.35 oe [not 65, 0.65 unless 1– 0.65] accept 2.86 or better (k= 2.85714... or $2\frac{6}{7}$ oe) [k = 2.9, k = 1/0.35 scores M1A0]																		
3	<table border="1" data-bbox="416 448 801 914"> <thead> <tr> <th>Year</th> <th>% discount</th> </tr> </thead> <tbody> <tr> <td>2007</td> <td>0</td> </tr> <tr> <td>2008</td> <td>30</td> </tr> <tr> <td>2009</td> <td>40</td> </tr> <tr> <td>2010</td> <td>50</td> </tr> <tr> <td>2011</td> <td>60</td> </tr> <tr> <td>2012</td> <td>65</td> </tr> <tr> <td>2013</td> <td>65</td> </tr> <tr> <td>Total</td> <td>310</td> </tr> </tbody> </table> <p data-bbox="353 938 723 970">310% of the premium is £3875</p> <p data-bbox="353 986 701 1058">The premium is $\frac{£3875 \times 100}{310}$</p> <p data-bbox="544 1198 645 1230">=£1250</p>	Year	% discount	2007	0	2008	30	2009	40	2010	50	2011	60	2012	65	2013	65	Total	310	B1 M1 A1 [3]	obtaining 310 or 3.1 oe [from adding all relevant terms ie $0+30+40+50+60+65+65=310$ or $0 +0.3+0.4+0.5+0.6+0.65+0.65= 3.1$ soi (with or without first zero term) or from $700- 100- 70- 60- 50- 40- 2 \times 35$ oe] 3875 ÷ their 3.1 even if one term was missing from addition but must come from attempt at the appropriate addition [ie an error in adding to 310 or an omission of one term, an inclusion of say an extra 65, or an addition of 100,70,60,...etc with subsequent subtraction from 700] £1250 cao
Year	% discount																				
2007	0																				
2008	30																				
2009	40																				
2010	50																				
2011	60																				
2012	65																				
2013	65																				
Total	310																				

Question	Answer	Marks	Guidance
4 (i)		<p>B1</p> <p>[1]</p>	<p>A sketch. Must be at least for values of x between 18 and 45. Must be correct shape ie descending curve, does not curve up at end, does not cross the horizontal axis (even if extended) ie must $\rightarrow 1$. Does not need to go through points exactly as a sketch.</p>
4 (ii)	<p>For large values of x, $be^{-k(x-17)} \rightarrow 0$, and so $y \rightarrow a$ which is 1 in this case.</p>	<p>B1</p> <p>[1]</p>	<p>Need x becoming large or $\rightarrow \infty$, exponential term $\rightarrow 0$ oe and $y \rightarrow a$, $a = 1$ or $y = 1, a = 1$ oe NOT just at $x = 45, y = 1$</p>
4 (iii)	<p>Substituting $x = 23 \Rightarrow y = 6.6$</p> <p>This is quite close to the observed value of $y = 6$.</p>	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>subst $x = 23$ and finding $y = 6.6$ (6.60115...) (or subst $y = 6$ and finding $x = 23.3$ (23.3097...)) Only accept comparing x or y, not coefficients (or close to $x = 23$) Accept say, $y = 6.6 \approx 6$. But not say at $x = 23, y \approx 6$ with no evaluation seen Accept if states, say, $y = 6.6$ which is not consistent with $y = 6$ oe</p>

Question		Answer	Marks	Guidance
5	(A)	With no more than 3 points on his licence the driver's premium is not altered: £520	B1	£520
	(B)	The driver now has $3 + 6 = 9$ points. The new premium is $£520 \times 2^{\frac{9}{6}} = £1470.78$	B1 [2]	£1470.78 or £1471 [but not £1470.80]

4754 Applications of Advanced Mathematics (C4)

General Comments:

This paper was of a similar standard to previous papers-or perhaps slightly more straightforward. The questions were accessible to all candidates but there were sufficient questions for the more able candidates to show their skills. The standard of work was quite high.

The comprehension was well understood and good marks were scored in this Paper. It was pleasing to see that candidates have improved in some areas particularly when:-

- Remembering to put a constant when integrating
- Reading questions carefully
- Giving more detail when trying to establish a given result
- Using brackets
- Giving exact answers when required

Some candidates still use rounded answers when going on to use their answers in subsequent calculations. Also, too many candidates used values to 3dp when calculating an answer to 3dp in the trapezium rule.

The trigonometry question 4 involving 'Show' caused problems for many. Some felt that they could divide one side of an equation by something, usually $\cos\alpha\cos\beta$, without doing so on both sides. The concept of 'equals' was not understood. There was a general lack of rigour in this question.

Candidates should be encouraged to change constants when multiplying through, say, by a value.

Some candidates still assume that showing that a vector is perpendicular to one vector in the plane is sufficient to show that it is a normal vector.

Quite a number of candidates failed to attempt some parts.

There did not appear to be a shortage of time for either Paper.

Comments on Individual Questions:

Paper A

Question 1

Most candidates understood the method of expressing the fraction in partial fractions. Many were completely successful and most errors were arithmetic. A few incorrectly used

$$\frac{A}{(2-x)} + \frac{B}{(4+x^2)}$$

Question 2

Much here depended upon the candidate's ability to factorise correctly. On too many occasions the factor was found to be 4 or $\frac{1}{4}$ instead of 8. The general method for expanding the binomial expansion was understood and the binomial coefficients were usually correct. Some who had factorised correctly then forgot to include the 8 at the final stage. The validity was often correct but was sometimes omitted and a variety of incorrect responses were also seen including $-\frac{1}{4} < x < \frac{1}{4}$.

Question 3

- (i) Many errors were seen here. In a number of cases the candidates were in degree mode. For others h was given incorrectly. Many others used the wrong formula and some substituted x values in the formula or omitted 0 from the formula. However, probably the most common error was giving the y values to 3dp and then using these to give a final answer correct to 3dp.
- (ii) This was a good discriminator as it really tested whether candidates understood how the trapezium rule estimates area. Some believed that it always underestimated or always overestimated.

Question 4

- (i) There were some very good solutions here when showing the two trigonometric expressions were equal. However, the majority were not successful. The most common overall error was not treating both sides of an equation equally. Too often only one side was changed.
A common starting point was

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\cos\beta} = 1 - \tan\alpha\tan\beta.$$
 This was then followed by a confused attempt at dividing by $\sec\alpha\sec\beta$. Candidates need to multiply 'top and bottom' by the same thing. Questions that involve 'Showing' need more rigour.
- (ii) This part was more successful provided that candidates wrote down the identity for $\sec^2\alpha$. There were, however, some long and confused attempts.
- (iii) Most candidates scored the first two marks here. Many failed to give the second solution of 150° .

Question 5

- (i) Scores here tended to be 4 or 1. Much depended upon whether the candidates used the product rule when finding dy/dt . The very common error was to state that $dy/dt = 2te^{2t}$. The majority of candidates did understand the general method of using $dy/dt = dx/dt \cdot dx/dt$. A number also failed to cancel their final answer, often leaving it as $\frac{e^2}{e^3}$.
- (ii) Most candidates found t correctly in terms of x and then substituted it into y . The simplification was not always then complete.

Question 6

Many successfully scored full marks when finding this volume of revolution. Common errors included failing to rearrange correctly for x (or x^2) and using the lower limit as 0. A few tried to use x limits in a y function, or tried to find $\int \pi y^2 dx$.

Question 7

- (i) Most candidates scored well in this part. Most understood how to find the lengths of the edges and the angle CAB. Some candidates used the cosine rule in this part but most used the scalar product. Finding the area of the triangle was less successful. Often, $\frac{1}{2}$ base X height was used with lengths AB and AC, or occasionally $\frac{1}{2} ab \cos C$.

- (ii) (A) Usually two marks were scored in this part, but some candidates only found the scalar product with one vector in the plane.
- (iii) (B) The equation of the plane $4x-3y+10z+12=0$ was usually found correctly. Some had i,j,k in their answer instead of x,y,z .
- (iv) Candidates usually gave the correct equation of the plane although sometimes ' r ' was omitted. The method for finding the point of intersection was usually understood but there were numerical errors. Many obtained full marks.
- (v) This was one of the least well answered questions on the paper. This was because candidates thought that the perpendicular height of the tetrahedron was 5, equal to the height CD.

Question 8

- (i) The method of separating the variables and integrating was more popular than verification, and was more successful. Those that verified usually forgot to use the initial conditions. When integrating there was sometimes confused work when the arbitrary constant was changed but continued to be used as c .
- (ii) Good marks were scored in this part by all candidates. Some made the question more difficult when finding A by using a quadratic equation. The most common error was in using $\sqrt{0.5}$ as 0.25 when finding t .
- (iii) A pleasing number of candidates scored full marks here. Most separated the variables correctly and successfully integrated the RHS, including the inclusion of $+c$. Those candidates who realised to expand the bracket and divide often were able to score all the remaining marks. A few used the approach from integration by parts but usually did not reach the end.
- (iv) Many good scores were achieved here when substituting to find B and t . There were a lot of numerical errors from others.

Paper B

Question 1

- (i) Most candidates correctly completed the table.
- (ii) $\pounds 24000/150=\pounds 160$ was usually correct. Some candidates multiplied or divided by additional values, usually 18 or 9.
- (iii) This was often correct but often long methods were seen including $\pounds 24000 \times 1.5 = \pounds 36000$, then 18.75% of $\pounds 36000 = \pounds 6750$ instead of the more direct approach of $\pounds 4500 \times 1.5 = \pounds 6750$. Some candidates stopped at $\pounds 6750$.

Question 2

This was not particularly well answered. Candidates seemed almost equally to find correctly that $k=1/0.35=2.86$ or to use $1/0.65$. A number of candidates used $1-0.65$ as 0.45 .

Question 3

This was a good discriminator on Paper B. There were many solutions scoring full marks. Others often included the wrong number of 0.65s in their total or sometimes incorrectly used $0.7+0.6+0.5+0.4\dots$ and failed to subtract.

Some others had no idea and tried to find say 65% of $\pounds 3875$.

Question 4

- (i) The graph was generally good. The commonest error was that it often stopped at $x=44$ instead of continuing to at least $x=45$. In other cases the graph looked as though it would cut through the x axis when produced.

- (ii) Few candidates were able to give a complete solution that as $x \rightarrow \infty, be^{-k(x-17)} \rightarrow 0$ and so as $y \rightarrow a, a = 1$ or equivalent. Many referred to the general shape of the curve or asymptotes or said that at $x=45, y=1$. Generally the responses were unclear or incomplete.
- (iii) This was well answered although some candidates forgot to compare the result of their calculation with the given point. Most compared 6 with 6.60... but some used 23. A few tried to establish the given constants b and k .

Question 5

Most candidates were successful here. A few did not round a money answer to an appropriate accuracy.

Unit level raw mark and UMS grade boundaries June 2014 series
AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)		Max Mark	a	b	c	d	e	u
4751/01 (C1) MEI Introduction to Advanced Mathematics	Raw	72	61	56	51	46	42	0
	UMS	100	80	70	60	50	40	0
4752/01 (C2) MEI Concepts for Advanced Mathematics	Raw	72	57	51	45	39	33	0
	UMS	100	80	70	60	50	40	0
4753/01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	47	42	36	0
4753/02 (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753 (C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01 (C4) MEI Applications of Advanced Mathematics	Raw	90	68	61	54	47	41	0
	UMS	100	80	70	60	50	40	0
4755/01 (FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	63	57	51	45	40	0
	UMS	100	80	70	60	50	40	0
4756/01 (FP2) MEI Further Methods for Advanced Mathematics	Raw	72	60	54	48	42	36	0
	UMS	100	80	70	60	50	40	0
4757/01 (FP3) MEI Further Applications of Advanced Mathematics	Raw	72	57	51	45	39	34	0
	UMS	100	80	70	60	50	40	0
4758/01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	0
4758/02 (DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758 (DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01 (M1) MEI Mechanics 1	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4762/01 (M2) MEI Mechanics 2	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4763/01 (M3) MEI Mechanics 3	Raw	72	55	48	42	36	30	0
	UMS	100	80	70	60	50	40	0
4764/01 (M4) MEI Mechanics 4	Raw	72	48	41	34	28	22	0
	UMS	100	80	70	60	50	40	0
4766/01 (S1) MEI Statistics 1	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0
4767/01 (S2) MEI Statistics 2	Raw	72	60	53	46	40	34	0
	UMS	100	80	70	60	50	40	0
4768/01 (S3) MEI Statistics 3	Raw	72	61	54	47	41	35	0
	UMS	100	80	70	60	50	40	0
4769/01 (S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4771/01 (D1) MEI Decision Mathematics 1	Raw	72	51	46	41	36	31	0
	UMS	100	80	70	60	50	40	0
4772/01 (D2) MEI Decision Mathematics 2	Raw	72	46	41	36	31	26	0
	UMS	100	80	70	60	50	40	0
4773/01 (DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
	UMS	100	80	70	60	50	40	0
4776/01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	54	48	43	38	32	0
4776/02 (NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82 (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776 (NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01 (NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
	UMS	100	80	70	60	50	40	0
4798/01 (FPT) Further Pure Mathematics with Technology	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0
GCE Statistics (MEI)		Max Mark	a	b	c	d	e	u
G241/01 (Z1) Statistics 1	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0
G242/01 (Z2) Statistics 2	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
G243/01 (Z3) Statistics 3	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0